Unit 1 - Lesson 15

An introduction to Complex Numbers

SWUT:

Imaginary numbers were introduced to solve quadratic equations of the form $x^{2}+k=0$

Where $k>0.$

The imaginary unit $i$ is a number defined as$ \sqrt{-1}$. Additionally$ i^{2}=-1$.

The powers of $i$ repeat in the cyclical pattern: $i, -1, -i, 1$

The set of complex numbers includes imaginary and real numbers. Every complex number can be written in the form $a+bi$, where $a$ represents the real part and $bi$ represents the imaginary part.

The modulus, or absolute value of a complex number, is defined as its distance from the origin in the complex plane.

In the complex number plane, (Argand plane), the point $(a, b)$ represents the complex number$ a+bi$. To graph a complex number, locate the real on the horizontal axis and the imaginary part on the vertical axis.

Complex conjugates are in the form of $a+bi$ and$ a-bi$.

Operations of addition, subtraction, multiplication, and division can be performed on complex numbers.

When performing operations on complex numbers, treat $i$ like a variable$ x$. Simplify powers of $i$ at the completion of the problem.

Simplifying Radicals

1. $\sqrt{32}$ 2. $-\sqrt{18}$ 3. $2\sqrt{200}$

4. $3\sqrt{12}+5\sqrt{27}$ 5. $\left(\sqrt{8}\right)\left(\sqrt{3}\right)$ 6. $\frac{\sqrt{48}}{\sqrt{6}}$

The imaginary unit $i$ is defined as

$$i=\sqrt{-1}$$

Similarly

$$i^{2}=-1$$

Basically for any positive number $a$ (***the negative comes out from the radical and becomes “i”***)

$$\sqrt{-a}=i\sqrt{a}$$

Try these

1. $\sqrt{-9}$ 2. $\sqrt{-7}$ 3. $\sqrt{-18}$

The Cyclic nature of $i$ How to simplify $i^{n}$



$$i^{0}=$$

$$i^{1}=$$

$$i^{2}=$$

$$i^{3}=$$

It is important to realize that multiplying two imaginary numbers can result in a real number!

**Operations with *i***

1. $i^{7}$ 2. $2i^{3}∙i^{11}$
2. $4i^{5}∙i^{15}$ 4. $12i^{13}+2i^{9}$
3. $\sqrt{-36}+\sqrt{-4}$ 6. $2\sqrt{-27}-3\sqrt{-75}$
4. $\sqrt{-9}∙\sqrt{-16}$ 8. $\sqrt{-4}∙\sqrt{-4}$

***Question:*** Does $\sqrt[3]{-8}$ result in imaginary numbers?

A whole new number system was formed called the Complex Number System



**Operations with Complex Numbers**

Simplify. Remember all answers must be written in $a+bi$ form.

1. $\left(4-3i\right)+\left(-6-2i\right)$ 2. $\left(5-3i\right)-\left(1+7i\right)$
2. $3\left(2+4i\right)-2\left(4-5i\right)$ 4. $(2+3i)(3+4i)$

$5. \left(2+5i\right)^{2}$ 6. $(3+5i)(3-5i)$

**Complex Conjugates**

Multiply each of the following complex numbers by its conjugate

1. $(3-4i)$ 2. $(-3-8i)$
2. $(9zi)$ 4. $(-4+15xi)$

Dividing complex numbers

1. $\frac{19}{i}$ 6. $\frac{9+3i}{4i}$
2. $\frac{4+5i}{-3i}$ 8. $\frac{-4-7i}{4-i}$

***Homework 1-15:***







